12.5 EXERCISES

1. Determine whether each statement is true or false.
(a) Two lines parallel to a third line are parallel.
(b) Two lines perpendicular to a third line are parallel.
(c) Two planes parallel to a third plane are parallel.
(d) Two planes perpendicular to a third plane are parallel.
(e) Two lines parallel to a plane are parallel.
(f) Two lines perpendicular to a plane are parallel.
(g) Two planes parallel to a line are parallel.
(h) Two planes perpendicular to a line are parallel.
(i) Two planes either intersect or are parallel.
(j) Two lines either intersect or are parallel.
(k) A plane and a line either intersect or are parallel.

2–5 Find a vector equation and parametric equations for the line.

2. The line through the point $(6, -5, 2)$ and parallel to the vector $\langle 1, 3, -\frac{2}{3} \rangle$

3. The line through the point $(2, 2.4, 3.5)$ and parallel to the vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

4. The line through the point $(0, 14, -10)$ and parallel to the line $x = -1 + 2t, y = 6 - 3t, z = 3 + 9t$

5. The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$

6–12 Find parametric equations and symmetric equations for the line.

6. The line through the origin and the point $(1, 2, 3)$

7. The line through the points $(1, 3, 2)$ and $(-4, 3, 0)$

8. The line through the points $(6, 1, -3)$ and $(2, 4, 5)$

9. The line through the points $(0, \frac{1}{2}, 1)$ and $(2, 1, -3)$

10. The line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$

11. The line through $(1, -1, 1)$ and parallel to the line $x + 2 = \frac{1}{2}y = z - 3$

12. The line of intersection of the planes $x + y + z = 1$ and $x + z = 0$

13. Is the line through $(-4, -6, 1)$ and $(-2, 0 -3)$ parallel to the line through $(10, 18, 4)$ and $(5, 3, 14)$?

14. Is the line through $(4, 1, -1)$ and $(2, 5, 3)$ perpendicular to the line through $(-3, 2, 0)$ and $(5, 1, 4)$?

15. (a) Find symmetric equations for the line that passes through the point $(1, -5, 6)$ and is parallel to the vector $\langle -1, 2, -3 \rangle$.
(b) Find the points in which the required line in part (a) intersects the coordinate planes.

16. (a) Find parametric equations for the line through $(2, 4, 6)$ that is perpendicular to the plane $x - y + 3z = 7$.
(b) In what points does this line intersect the coordinate planes?

17. Find a vector equation for the line segment from $(2, -1, 4)$ to $(4, 6, 1)$.

18. Find parametric equations for the line segment from $(10, 3, 1)$ to $(5, 6, -3)$.

19–22 Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

19. L_1: $x = -6t, \quad y = 1 + 9t, \quad z = -3t$
L_2: $x = 1 + 2s, \quad y = 4 - 3s, \quad z = s$

20. L_1: $x = 1 + 2t, \quad y = 3t, \quad z = 2 - t$
L_2: $x = -1 + s, \quad y = 4 + s, \quad z = 1 + 3s$

21. L_1: $\dfrac{x}{1} = \dfrac{y - 1}{2} = \dfrac{z - 2}{3}$
L_2: $\dfrac{x - 3}{-4} = \dfrac{y - 2}{-3} = \dfrac{z - 1}{2}$

22. L_1: $\dfrac{x - 1}{2} = \dfrac{y - 3}{2} = \dfrac{z - 2}{-1}$
L_2: $\dfrac{x - 2}{1} = \dfrac{y - 6}{-1} = \dfrac{z + 2}{3}$

23–38 Find an equation of the plane.

23. The plane through the point $(6, 3, 2)$ and perpendicular to the vector $\langle -2, 1, 5 \rangle$

24. The plane through the point $(4, 0, -3)$ and with normal vector $\mathbf{j} + 2\mathbf{k}$

25. The plane through the point $(1, -1, 1)$ and with normal vector $\mathbf{i} + \mathbf{j} - \mathbf{k}$

26. The plane through the point $(-2, 8, 10)$ and perpendicular to the line $x = 1 + t, y = 2t, z = 4 - 3t$

27. The plane through the origin and parallel to the plane $2x - y + 3z = 1$

28. The plane through the point $(-1, 6, -5)$ and parallel to the plane $x + y + z + 2 = 0$

29. The plane through the point $(4, -2, 3)$ and parallel to the plane $3x - 7z = 12$

30. The plane that contains the line $x = 3 + 2t, y = t, z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$

31. The plane through the points $(0, 1, 1), (1, 0, 1)$, and $(1, 1, 0)$

32. The plane through the origin and the points $(2, -4, 6)$ and $(5, 1, 3)$

33. The plane through the points $(3, -1, 2)$, $(8, 2, 4)$, and $(-1, -2, -3)$

34. The plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$

35. The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$

36. The plane that passes through the point $(1, -1, 1)$ and contains the line with symmetric equations $x = 2y = 3z$

37. The plane that passes through the point $(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$

38. The plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and is perpendicular to the plane $x + y - 2z = 1$

39–42 Use intercepts to help sketch the plane.

39. $2x + 5y + z = 10$

40. $3x + y + 2z = 6$

41. $6x - 3y + 4z = 6$

42. $6x + 5y - 3z = 15$

43–45 Find the point at which the line intersects the given plane.

43. $x = 3 - t,\ y = 2 + t,\ z = 5t;\quad x - y + 2z = 9$

44. $x = 1 + 2t,\ y = 4t,\ z = 2 - 3t;\quad x + 2y - z + 1 = 0$

45. $x = y - 1 = 2z;\quad 4x - y + 3z = 8$

46. Where does the line through $(1, 0, 1)$ and $(4, -2, 2)$ intersect the plane $x + y + z = 6$?

47. Find direction numbers for the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$.

48. Find the cosine of the angle between the planes $x + y + z = 0$ and $x + 2y + 3z = 1$.

49–54 Determine whether the planes are parallel, perpendicular, or neither. If neither, find the angle between them.

49. $x + 4y - 3z = 1,\quad -3x + 6y + 7z = 0$

50. $2z = 4y - x,\quad 3x - 12y + 6z = 1$

51. $x + y + z = 1,\quad x - y + z = 1$

52. $2x - 3y + 4z = 5,\quad x + 6y + 4z = 3$

53. $x = 4y - 2z,\quad 8y = 1 + 2x + 4z$

54. $x + 2y + 2z = 1,\quad 2x - y + 2z = 1$

55–56 (a) Find parametric equations for the line of intersection of the planes and (b) find the angle between the planes.

55. $x + y + z = 1,\quad x + 2y + 2z = 1$

56. $3x - 2y + z = 1,\quad 2x + y - 3z = 3$

57–58 Find symmetric equations for the line of intersection of the planes.

57. $5x - 2y - 2z = 1,\quad 4x + y + z = 6$

58. $z = 2x - y - 5,\quad z = 4x + 3y - 5$

59. Find an equation for the plane consisting of all points that are equidistant from the points $(1, 0, -2)$ and $(3, 4, 0)$.

60. Find an equation for the plane consisting of all points that are equidistant from the points $(2, 5, 5)$ and $(-6, 3, 1)$.

61. Find an equation of the plane with x-intercept a, y-intercept b, and z-intercept c.

62. (a) Find the point at which the given lines intersect:

$$\mathbf{r} = \langle 1, 1, 0 \rangle + t\langle 1, -1, 2 \rangle$$
$$\mathbf{r} = \langle 2, 0, 2 \rangle + s\langle -1, 1, 0 \rangle$$

(b) Find an equation of the plane that contains these lines.

63. Find parametric equations for the line through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$.

64. Find parametric equations for the line through the point $(0, 1, 2)$ that is perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$ and intersects this line.

65. Which of the following four planes are parallel? Are any of them identical?

P_1: $4x - 2y + 6z = 3$ $\qquad$ P_2: $4x - 2y - 2z = 6$

P_3: $-6x + 3y - 9z = 5$ $\qquad$ P_4: $z = 2x - y - 3$

66. Which of the following four lines are parallel? Are any of them identical?

L_1: $x = 1 + t,\quad y = t,\quad z = 2 - 5t$

L_2: $x + 1 = y - 2 = 1 - z$

L_3: $x = 1 + t,\quad y = 4 + t,\quad z = 1 - t$

L_4: $\mathbf{r} = \langle 2, 1, -3 \rangle + t\langle 2, 2, -10 \rangle$

67–68 Use the formula in Exercise 43 in Section 12.4 to find the distance from the point to the given line.

67. $(4, 1, -2);\quad x = 1 + t,\ y = 3 - 2t,\ z = 4 - 3t$

68. $(0, 1, 3);\quad x = 2t,\ y = 6 - 2t,\ z = 3 + t$

69–70 Find the distance from the point to the given plane.

69. $(1, -2, 4),\quad 3x + 2y + 6z = 5$

70. $(-6, 3, 5),\quad x - 2y - 4z = 8$

71–72 Find the distance between the given parallel planes.

71. $2x - 3y + z = 4,\quad 4x - 6y + 2z = 3$

72. $6z = 4y - 2x, \quad 9z = 1 - 3x + 6y$

73. Show that the distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

74. Find equations of the planes that are parallel to the plane $x + 2y - 2z = 1$ and two units away from it.

75. Show that the lines with symmetric equations $x = y = z$ and $x + 1 = y/2 = z/3$ are skew, and find the distance between these lines.

76. Find the distance between the skew lines with parametric equations $x = 1 + t$, $y = 1 + 6t$, $z = 2t$, and $x = 1 + 2s$, $y = 5 + 15s$, $z = -2 + 6s$.

77. If a, b, and c are not all 0, show that the equation $ax + by + cz + d = 0$ represents a plane and $\langle a, b, c\rangle$ is a normal vector to the plane.

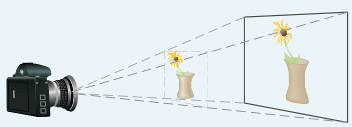
Hint: Suppose $a \neq 0$ and rewrite the equation in the form

$$a\left(x + \frac{d}{a}\right) + b(y - 0) + c(z - 0) = 0$$

78. Give a geometric description of each family of planes.
(a) $x + y + z = c$ (b) $x + y + cz = 1$
(c) $y\cos\theta + z\sin\theta = 1$

LABORATORY PROJECT

PUTTING 3D IN PERSPECTIVE



Computer graphics programmers face the same challenge as the great painters of the past: how to represent a three-dimensional scene as a flat image on a two-dimensional plane (a screen or a canvas). To create the illusion of perspective, in which closer objects appear larger than those farther away, three-dimensional objects in the computer's memory are projected onto a rectangular screen window from a viewpoint where the eye, or camera, is located. The viewing volume—the portion of space that will be visible—is the region contained by the four planes that pass through the viewpoint and an edge of the screen window. If objects in the scene extend beyond these four planes, they must be truncated before pixel data are sent to the screen. These planes are therefore called *clipping planes*.

1. Suppose the screen is represented by a rectangle in the yz-plane with vertices $(0, \pm 400, 0)$ and $(0, \pm 400, 600)$, and the camera is placed at $(1000, 0, 0)$. A line L in the scene passes through the points $(230, -285, 102)$ and $(860, 105, 264)$. At what points should L be clipped by the clipping planes?
2. If the clipped line segment is projected on the screen window, identify the resulting line segment.
3. Use parametric equations to plot the edges of the screen window, the clipped line segment, and its projection on the screen window. Then add sight lines connecting the viewpoint to each end of the clipped segments to verify that the projection is correct.
4. A rectangle with vertices $(621, -147, 206)$, $(563, 31, 242)$, $(657, -111, 86)$, and $(599, 67, 122)$ is added to the scene. The line L intersects this rectangle. To make the rectangle appear opaque, a programmer can use *hidden line rendering*, which removes portions of objects that are behind other objects. Identify the portion of L that should be removed.

12.6 CYLINDERS AND QUADRIC SURFACES

We have already looked at two special types of surfaces: planes (in Section 12.5) and spheres (in Section 12.1). Here we investigate two other types of surfaces: cylinders and quadric surfaces.

In order to sketch the graph of a surface, it is useful to determine the curves of intersection of the surface with planes parallel to the coordinate planes. These curves are called **traces** (or cross-sections) of the surface.